Mathematics 232 Fall 1998 Exam 3

November 3, 1998

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Name

Technology used: ____

Textbook/Notes used: ____

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. **Only write on one side of each page.**

The Problems

1. Let T and L be transformations from R^n to R^n. Suppose that L is the inverse of T. If T is a **linear** transformation, show that L is closed under addition. (That is, show that L satisfies the first part of **Fact 2.2.1.**)

2. The plane $3x + 2y + z = 0$ is a subspace, V , of R^3.

   (a) Find a matrix A so that $V = \ker(A)$.

   (b) Find a matrix B so that $V = \mathrm{Im}(B)$.

3. Let V be the subspace of R^4 spanned by $\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$. Find a basis for $V^\perp$.

4. What is an **orthonormal basis** for $V^\perp$ in the previous question?

5. Suppose $\overrightarrow{v}_1, \overrightarrow{v}_2, \overrightarrow{v}_3$ is a linearly independent set in R^5. Is the set of vectors $2\overrightarrow{v}_1 + \overrightarrow{v}_2 + 3\overrightarrow{v}_3,\ \overrightarrow{v}_2 + 5\overrightarrow{v}_3,\ 3\overrightarrow{v}_1 + \overrightarrow{v}_2 + 2\overrightarrow{v}_3$ linearly dependent or independent?

6. Let V and W be two subspaces of R^n. Show the intersection $V \cap W$ is also a subspace?

7. Let V and W be two subspaces of R^n. Define
$$V + W = \{\overrightarrow{v} + \overrightarrow{w} \in R^n : \overrightarrow{v} \in V \text{ and } \overrightarrow{w} \in W\}.$$
   Show that $V + W$ is a subspace of R^n.

8. Let $\overrightarrow{v}_1, \dots, \overrightarrow{v}_m$ be a basis for a subspace V of R^n. Show that if $\overrightarrow{x} \in R^n$ satisfies
$$\overrightarrow{v}_i \cdot \overrightarrow{x} = 0, \quad \text{for all } i = 1, \dots, m$$
   then $\overrightarrow{x} \in V^\perp$. That is, $\overrightarrow{x}$ is perpendicular to **every** vector in V.

9. Is there an orthogonal linear transformation $T : R^3 \to R^3$ for which
$$T \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} \quad \text{and} \quad T \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}?$$

10. If A is an $n \times n$ symmetric, invertible matrix must A^{-1} also be symmetric?

11. Suppose $T : R^n \rightarrow R^m$ is a linear transformation with $\ker(T) = \{\overrightarrow{0}\}$. Show that if $\overrightarrow{v}_1, \overrightarrow{v}_2, \overrightarrow{v}_3$ are linearly independent in R^n then $T(\overrightarrow{v}_1), T(\overrightarrow{v}_2), T(\overrightarrow{v}_3)$ are also linearly independent.